Ohm's and Kirchhoff's Laws: Simple Circuits

Introduction

The circuits in this set of problems are called simple circuits because

1. Each circuit consists of a small number of circuit elements.
2. Each circuit consists only of resistors, voltage sources, current sources, and either a voltmeter or ammeter.
3. The voltages of the voltage sources and currents of the current sources have constant values.

The inputs to these circuits are the voltages of the voltage sources and/or the currents of the current sources. The output of each circuit is either a voltage measured by a voltmeter or a current measured by an ammeter. All of the inputs have constant values. Consequently, the outputs also have constant values.

Use Ohm's and Kirchhoff's laws to solve these problems. In particular, it's important to notice that Ohm's law applies to a current and voltage that adhere to the passive convention.

Ohm's law is discussed in Section 2.6 of *Introduction to Electric Circuits* by R.C. Dorf and J.A Svoboda. Kirchhoff's laws are discussed in Section 3.3. Voltage and current sources are described in Section 2.6. Voltmeters and ammeters are described in Section 2.7. The passive convention is described in Section 1.6.

Worked Examples

Example 1:

Consider the circuit shown in Figure 1. Find the value of the resistance, *R*. Find the power *supplied by* each source and the power *absorbed by* the resistor.

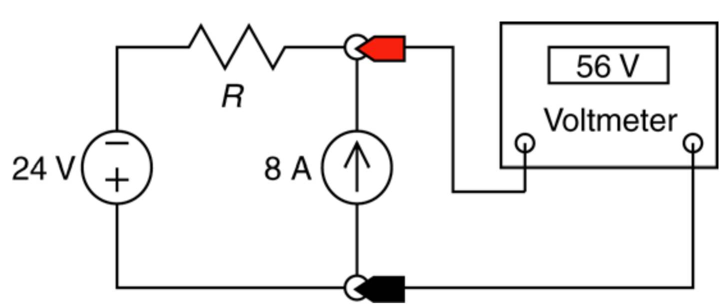

Figure 1 The circuit considered in Example 1.

Solution: Figure 2 shows the circuit from Figure 1 after replacing the voltmeter by an equivalent open circuit and labeling the voltage measured by the voltmeter.

Label the element currents and voltages as shown in Figure 3. In anticipation of using Ohm's law, the reference directions of the resistor current and voltage have been chosen to adhere to the passive convention. Consequently, the product of the resistor current and voltage is the power *absorbed by* the resistor, as required.

The reference direction for the voltage source current has been selected so that the voltage source current and voltage do not adhere to the passive convention. Consequently, the product of the voltage source current and voltage is the power *supplied by* the voltage source, as required.

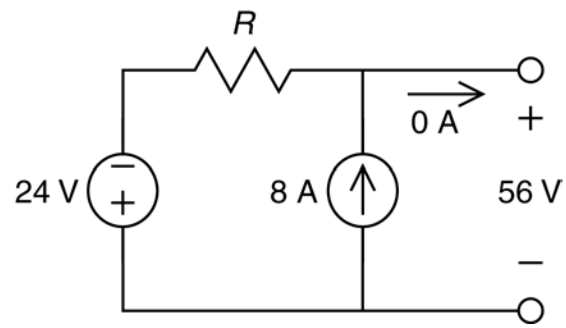

Figure 2 The circuit from Figure 1 after replacing the voltmeter by an open circuit.

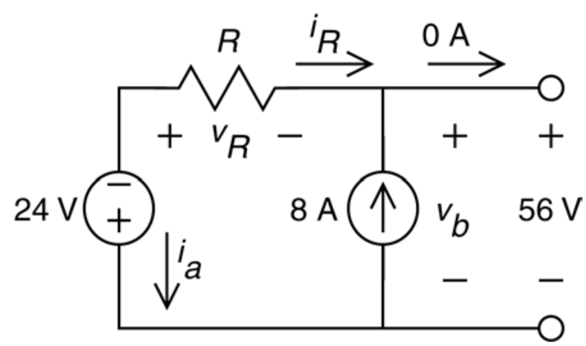

Figure 3 The circuit from Figure 2 after labeling the element currents and voltages.

Similarly, the reference direction for the current source voltage has been selected so that the current source current and voltage do not adhere to the passive convention. Consequently, the product of the current source current and voltage is the power *supplied by* the current source, as required.

Apply KVL to the loop consisting of the voltage source, the resistor and the open circuit that replaced to voltmeter to get

$$v_R + 56 + 24 = 0 \quad \Rightarrow \quad v_R = -80 \text{ V}$$

Apply KCL at the top node of the current source to get

$$i_R + 8 = 0 \quad \Rightarrow \quad i_R = -8 \text{ A}$$

Next, Ohm's law gives

$$R = \frac{v_R}{i_R} = \frac{-80}{-8} = 10 \ \Omega$$

The power *absorbed by* the resistor is

$$i_R \ v_R = (-8)(-80) = 640 \text{ W}$$

Apply KCL at the node that connects the voltage source and resistor to get

$$0 = i_a + i_R \quad \Rightarrow \quad i_a = -i_R = 8 \text{ A}$$

The power *supplied by* the voltage source is

$$i_a \,(24) = (8)(24) = 192 \text{ W}$$

Apply KVL to the mesh consisting of the current source and the open circuit that replaced to voltmeter to get

$$56 - v_b = 0 \quad \Rightarrow \quad v_b = 56 \text{ V}$$

The power *supplied by* the current source is

$$(8)\, v_b = (8)(56) = 448 \text{ W}$$

As expected, the power absorbed by the resistor is equal the sum of the powers delivered by the sources.

Example 2:

Consider the circuit shown in Figure 4. Find the value of the resistance, *R*. Find the value of the power *supplied by* the voltage source and the power *absorbed by* the resistor.

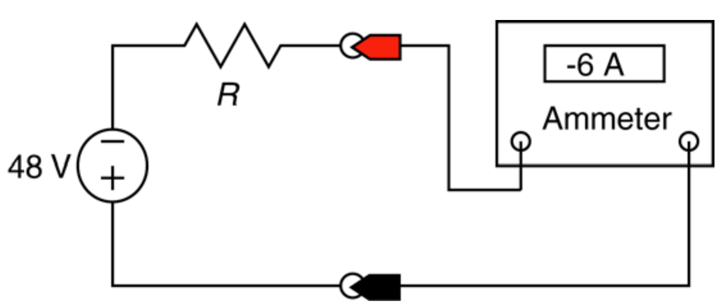

Figure 4 The circuit considered in Example 2.

Solution: Figure 5 shows the circuit from Figure 4 after replacing the ammeter by an equivalent short circuit and labeling the current measured by the ammeter.

Label the element currents and voltages as shown in Figure 6. In anticipation of using Ohm's law, the reference directions of the resistor current and voltage have been chosen to adhere to the passive convention. Consequently, the product of the resistor current and voltage is the power *absorbed by* the resistor, as required.

The reference direction for the voltage source current has been selected so that the voltage source current and voltage do not adhere to the passive convention. Consequently, the product of the voltage source current and voltage is the power *supplied by* the voltage source, as required.

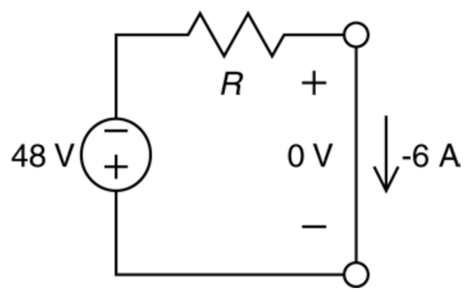

Figure 5 The circuit from Figure 4 after replacing the ammeter by a short circuit.

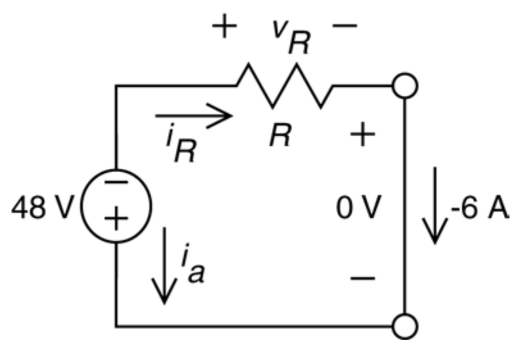

Figure 6 The circuit from Figure 5 after labeling the element currents and voltages.

Apply KVL to the mesh consisting of the voltage source, the resistor and the short circuit that replaced to ammeter to get

$$v_R + 0 + 48 = 0 \quad \Rightarrow \quad v_R = -48 \text{ V}$$

Apply KCL at the right node of the resistor to get

$$i_R = -6 \quad \Rightarrow \quad i_R = -6 \text{ A}$$

Next, Ohm's law gives

$$R = \frac{v_R}{i_R} = \frac{-48}{-6} = 8\ \Omega$$

The power *absorbed by* the resistor is

$$i_R\ v_R = (-6)(-48) = 288\ \text{W}$$

Apply KCL at the node that connects the voltage source and resistor to get

$$0 = i_a + i_R \quad \Rightarrow \quad i_a = -i_R = 6\ \text{A}$$

The power *supplied by* the voltage source is

$$i_a\,(48) = (6)(48) = 288\ \text{W}$$

As expected, the power absorbed by the resistor is equal the power delivered by the voltage source.

Example 3:

Consider the circuit shown in Figure 7. Find the value of the resistance, *R*. Find the value of the power *supplied by* each source and the power *absorbed by* the resistor.

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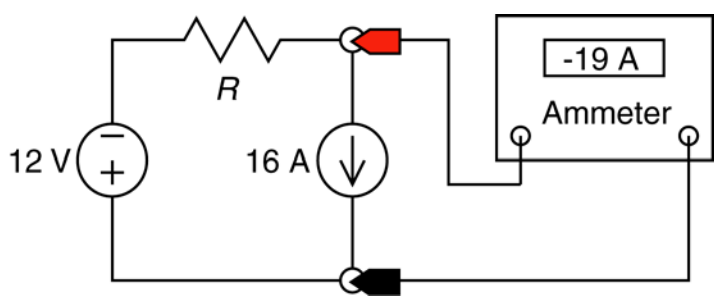

Figure 7 The circuit considered in Example 3.

Solution: Figure 8 shows the circuit from Figure 7 after replacing the ammeter by an equivalent short circuit and labeling the current measured by the ammeter.

Label the element currents and voltages as shown in Figure 9. In anticipation of using Ohm's law, the reference directions of the resistor current and voltage have been chosen to adhere to the passive convention. Consequently, the product of the resistor current and voltage is the power *absorbed by* the resistor, as required.

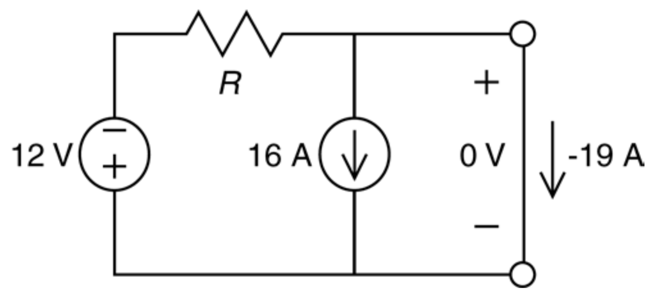

Figure 8 The circuit from Figure 7 after replacing the ammeter by a short circuit.

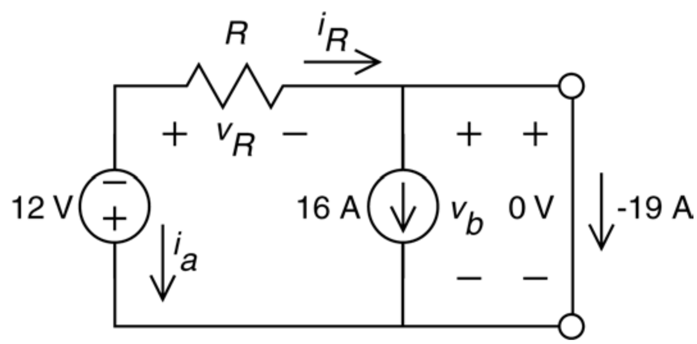

Figure 9 The circuit from Figure 8 after labeling the element currents and voltages.

The reference direction for the voltage source current has been selected so that the voltage source current and voltage do not adhere to the passive convention. Consequently, the product of the voltage source current and voltage is the power *supplied by* the voltage source, as required.

Similarly, the reference direction for the current source voltage has been selected so that the current source current and voltage do not adhere to the passive convention. Consequently, the product of the current source current and voltage is the power *supplied by* the current source, as required.

Apply KVL to the loop consisting of the voltage source, the resistor and the short circuit that replaced to ammeter to get

$$v_R + 0 + 12 = 0 \quad \Rightarrow \quad v_R = -12 \text{ V}$$

Apply KCL at the top node of the current source to get

$$i_R = 16 + (-19) \quad \Rightarrow \quad i_R = -3 \text{ A}$$

Next, Ohm's law gives

$$R = \frac{v_R}{i_R} = \frac{-12}{-3} = 4 \ \Omega$$

The power *absorbed by* the resistor is

$$i_R\, v_R = (-3)(-12) = 36 \text{ W}$$

Apply KCL at the node that connects the voltage source and resistor to get

$$0 = i_a + i_R \quad \Rightarrow \quad i_a = -i_R = 3 \text{ A}$$

The power *supplied by* the voltage source is

$$i_a\,(12) = (3)(12) = 36 \text{ W}$$

Apply KVL to the mesh consisting of the current source and the short circuit that replaced to ammeter to get

$$0 - v_b = 0 \quad \Rightarrow \quad v_b = 0 \text{ V}$$

The power *supplied by* the current source is

$$(16)\, v_b = (16)(0) = 0 \text{ W}$$

As expected, the power absorbed by the resistor is equal the sum of the powers delivered by the sources.

Example 4:
Consider the circuit shown in Figure 10. Find the value of the resistance, *R*. Find the value of the power *supplied by* the current source and the power *absorbed by* the resistor.

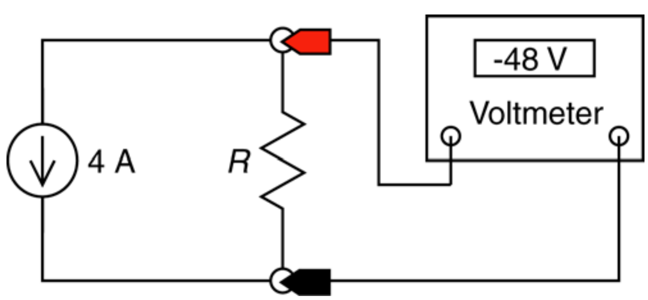

Figure 10 The circuit considered in Example 4.

Solution: Figure 11 shows the circuit from Figure 10 after replacing the voltmeter by an equivalent open circuit and labeling the voltage measured by the voltmeter.

Label the element currents and voltages as shown in Figure 12. In anticipation of using Ohm's law, the reference directions of the resistor current and voltage have been chosen to adhere to the passive convention. Consequently, the product of the resistor current and voltage is the power *absorbed by* the resistor, as required.

The reference direction for the current source voltage has been selected so that the current source current and voltage do not adhere to the passive convention. Consequently, the product of the current source current and voltage is the power *supplied by* the current source, as required.

Apply KVL to the mesh consisting of the resistor and the open circuit that replaced to voltmeter to get

$$-48 - v_R = 0 \quad \Rightarrow \quad v_R = -48 \text{ V}$$

Apply KCL at the top node of the resistor to get

$$0 + i_R + 4 = 0 \quad \Rightarrow \quad i_R = -4 \text{ A}$$

Next, Ohm's law gives

$$R = \frac{v_R}{i_R} = \frac{-48}{-4} = 12 \ \Omega$$

The power *absorbed by* the resistor is

$$i_R \ v_R = (-4)(-48) = 192 \text{ W}$$

Apply KVL to the mesh consisting of the current source and the resistor to get

$$v_R + v_a = 0 \quad \Rightarrow \quad v_a = -v_R = 48 \text{ A}$$

The power *supplied by* the voltage source is

$$(4)\, v_a = (4)(48) = 192 \text{ W}$$

As expected, the power absorbed by the resistor is equal the power delivered by the current source.

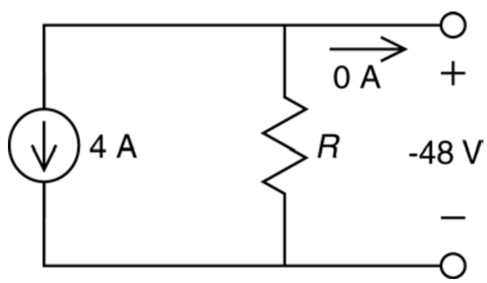

Figure 11 The circuit from Figure 10 after replacing the voltmeter by an open circuit.

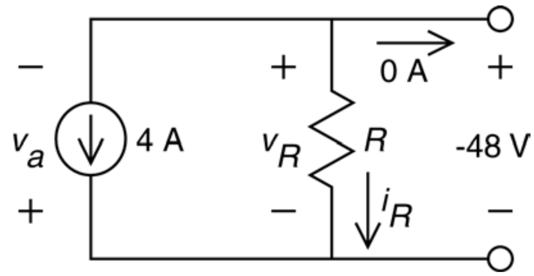

Figure 12 The circuit from Figure 11 after labeling the element currents and voltages.